Production/Maintenance Control of Multiple-Product Manufacturing System

Iahcen Mifdal
International University of Agadir
Agadir, Morocco
mifdal@e-polytechnique.ma

Zied Hajej / Sofiene Dellagi
LGIPM-Lorraine University, Metz, France
zied.hajej@univ-lorraine.fr
Sofiene.dellagi@univ-lorraine.fr

Abstract—This paper presents a method to find the optimal production rate and preventive maintenance policies for a multiple-product manufacturing system. The manufacturing system under consideration consists of one machine which produces several products in order to satisfy randomly demands corresponding to every product. The machine is subject to random failures. A preventive maintenance action is considered in order to improve the reliability of the machine, thereby reducing the amount of degradation caused by machine failures. The decision variables are the production rate, the sub-periods of production and the preventive maintenance rate. The objective of the study is to find the decision variables that minimize the overall cost, including production, preventive maintenance and inventory holding costs over a finite planning horizon. A numerical example is given to illustrate the proposed model.

Keywords—Preventive maintenance; production rate; multiple-product system; failure rate; random demand

I. INTRODUCTION

Recent years have seen considerable growth in interest in production planning and preventive maintenance control in manufacturing systems. In this context, [1] studied the control of corrective and preventive maintenance rates in the production planning of manufacturing system with machines subject to random failures and repairs. The objective of their study consists on minimizing a discounted overall cost consisting of maintenance cost, inventory holding and backlog cost. The study of [2] addresses the problem of finding robust production and maintenance schedules for a single machine with failures uncertainty. They propose a proactice joint model which simultaneously determines the production scheduling and maintenance policy to optimize the robustness of schedules. [3] developed a method to find the optimal production, replacement/repair and preventive maintenance policies for a degraded manufacturing system. [4] presented a joint strategy of buffer stock production and preventive maintenance for a randomly failing production unit operating in an environment where repair and preventive maintenance durations are random. Several reviews have been published to summarize the development in this area [5], [6] and [7].

This paper deals with the control problem of a stochastic manufacturing system consisting one machine producing several products. The stochastic nature of the system is due to the fact that the machine is subject to random failures and repairs. The production and maintenance control was studied by several researchers. [8] presented the analysis of the production control and corrective maintenance rate problem in a multiple-machine, multiple-product manufacturing system. They obtained a near optimal control policy of the system through numerical techniques by controlling both production and repair rates. [9] developed a stochastic dynamic optimization model to solve a multi-product, multi-period production planning problem with constraints on decision variables and finite planning horizon. [10] presented a Markov decision process model that simultaneously determines maintenance and production schedules for a multiple-product, single-machine production system, accounting for the fact that equipment condition can affect the yield of different product types differently. [11] developed a multi-product manufacturing systems problem with sequence dependent setup times and finite buffers under seven scheduling policies. Many works was developed in the same context, [12] and [13].

In the literature the consideration of the material degradation according to the production rate has been rarely studied. The present study takes into account the influence of the production planning on the material degradation in order to establish an optimal maintenance strategy. Thus, the system degradation depends not only on time but also on the production rate. In their study, [14], [15], [16] also took into consideration the influence of production plan on the equipment degradation in the case when one single machine is producing a single product type. In the same vein, [17] proposed a model where the failure rate of a machine depends on its age; hence, the corrective and preventive maintenance policies are machine-age dependent.

The reminder of this article is organized as follows: In the next section, we define notations and assumptions used in the developed models. In Section 3, we develop the production policy. The maintenance strategy is stated in section 4. A numerical example is presented in chapter 5. The paper is finally concluded in Section 6.

II. NOTATIONS AND ASSUMPTIONS

Throughout this article, we use the following notations and assumptions:

A. Notations

The following notations will be used to describe the proposed control model:
Assumptions

The mathematical model in this analysis is based on the following assumptions:

- We have divided the period $k$ into $p$ equal sub-periods, with $n = p$ (the total number of products);
- Holding and production costs of each product are known and constant;
- Only a single product can be produced in each sub-period;
- Maintenance actions have negligible durations;
- In the case of preventive maintenance, the system becomes as good as new;
- $Mp$ and $Mc$ costs incurred by the preventive and corrective maintenance actions are known and constant, with $Mc >> Mp$.

- The standard deviation of demand $\forall (d_i)$ and the average demand $d_i$ for each product and each period $k$ are known and constant. These two data allow us to obtain $d_i(k)$, the demand of each product in each period.

III. PRODUCTION POLICY

A. Problem formulation

In this section, we consider that the finite production horizon is divided into sub-periods. At any given sub-period, the machine can only produce one type of product. The objective is to determine a production plan for a finite time horizon $U^*(\{\ldots \{ \ldots \{ \ldots \{ \ldots \} \ldots \} \ldots \} \ldots \})$. The production plan must satisfy random demands under the requirement of a given level of service, while minimizing the cost of production and storage. Figure below shows an example of a production plan.

![Fig. 1. Repartition of the production plan](image)

The mathematical formulation of the proposed problem is based on the extension of the model described by [16] for the one product case study.

Formally, the stochastic production problem is defined as follows:

$$
\begin{align*}
U^*(\{\ldots \{ \ldots \{ \ldots \{ \ldots \} \ldots \} \ldots \} \ldots \}) & & \text{With: } \nonumber \\
\{\ldots \{ \ldots \{ \ldots \{ \ldots \} \ldots \} \ldots \} \ldots \}
\end{align*}
$$

$$
\begin{align*}
&\text{To develop this model, we based on the model developed by [17]. The difference is that the cost of storage is taken into account at the end of sub-periods, not at the end of periods.} \\
&\text{Therefore, the total cost of storage and production is defined as follows:} \\
&\begin{align*}
&\sum \{\ldots \{ \ldots \{ \ldots \{ \ldots \} \ldots \} \ldots \} \ldots \}
\end{align*}
\end{align*}
$$

Thus, the optimization problem is expressed as:

$$
\begin{align*}
&\sum \{\ldots \{ \ldots \{ \ldots \{ \ldots \} \ldots \} \ldots \} \ldots \}
\end{align*}
$$

Under the following constraints:!!!!!!!
So we can deduce:

\[ \left( \left( \prod_{i=1}^{n} x_{i} \right) \right) \]  

Reference

The deterministic production model

The equations (2) and (3) can be produced in sub-period \( j \) of period \( k \). The constraint (6) states that \( y_{ijk} \) is a binary variable. We note that \( \prod x_{i} \) is equal to 1 if the product \( i \) is produced in sub-period \( j \) of the period \( k \), and 0 otherwise.

B. The deterministic production model

The resolution of the stochastic problem under these assumptions is generally difficult. Thus, its transformation into a deterministic problem facilitates its resolution.

Reference [16] proved that:

\[ \left( \frac{1}{2} - \prod \right) \]  

\[ \prod \prod \]  

Where \( \tilde{S}(k) \) mean stock level at the end of period \( k \) and \( d \) the standard deviation of demand.

So we can deduce:

\[ \left\{ \left( \prod_{i=1}^{n} x_{i} \right) \right\} \]  

Using (9) and (10), we can determine the deterministic model of our stochastic problem (PI).

We know that:

\[ \left( \frac{1}{2} - \prod \right) \]  

Therefore, the deterministic equation of production/inventory cost is as follows:

**Lemma 1:**

\[ \left( \frac{1}{2} - \prod \right) \]  

The inventory balance equation

We know that: \( \left( \frac{1}{2} - \prod \right) \) and \( \prod \prod \prod \) \( \prod \prod \prod \prod \) and \( \prod \prod \prod \prod \) \( \prod \prod \prod \prod \)

Also:

\[ \left( \frac{1}{2} - \prod \right) \]  

Then:

\[ \left( \frac{1}{2} - \prod \right) \]  

And:

\[ \left( \frac{1}{2} - \prod \right) \]  

Therefore, the deterministic balance equation is defined as:

\[ \left( \frac{1}{2} - \prod \right) \]  


**The service level constraint:**

We have: 
\[ P = \Phi^*(1 - \frac{\mu}{\sigma}) \]

Then:
\[ \sum_{i=1}^{n} P_i = \sum_{i=1}^{n} \left( \Phi^*(1 - \frac{\mu_i}{\sigma_i}) \right) \]  
(13)

With:
\[ P_i = \Phi^*(1 - \frac{\mu_i}{\sigma_i}) \]

\[ U_i : \text{Minimum cumulative production quantity for each product } i \]

\[ \sigma_i : \text{Variance of demand } d_i \text{ at period } k. \]

\# Cumulative Gaussian distribution function with mean \( \mu \) and finite variance \( \sigma \).

\[ \Phi^* \] Inverse distribution function

Proof: (contact the author)

Constraints (3), (4), (5) and (6) are unchanged.

- In summary:

The deterministic optimization problem becomes:

Objective function:
\[
\min \left\{ \sum_{i=1}^{n} \sum_{j=1}^{m} \left( \int_0^\infty \left( \sum_{k=1}^{r} \sum_{l=1}^{s} \# \left( \frac{\mu_k}{\sigma_k} \right) \right) \right) \right\} 
\]

Under the constraints below:
\[
\sum_{i=1}^{n} \sum_{j=1}^{m} \left( \int_0^\infty \left( \sum_{k=1}^{r} \sum_{l=1}^{s} \# \left( \frac{\mu_k}{\sigma_k} \right) \right) \right) \right\} 
\]

IV. MAINTENANCE STRATEGY

A. Description

The maintenance strategy adopted in this study is known as preventive maintenance with minimal repair. More specifically, the start of our machine is applied to a horizon \( q \) \( T \) \( (q = 1, 2 \ldots) \). The actions of preventive maintenance are practiced in the period \( q \) \( T \) \( (q = 1, 2 \ldots) \). The replacement rule for this policy is to replace the system with another new system at each period \( q \) \( T \). At each failure between replacements or preventive maintenance actions, only one minimal repair is implemented.

If \( \phi(t) \) represents the function the failure rate, the average total cost of maintenance is expressed as following:

\[ \phi(t) \frac{\lambda}{\mu} \]

The existence of an optimal number of partitions \( N^* \), and therefore, the optimal preventive maintenance period \( T^* \) is proven in the literature. It has been proven that \( T^* \) exists if the failure rate is increasing \[ 19 \).

In this policy, the average total cost of maintenance depends on the production plan defined by \( \left( \begin{array} {ccc} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{array} \right) \]

and is expressed as:

\[ \frac{\lambda}{\mu} \]

B. The failure rate expression

To obtain the failure rate expression, we have taken into account the influence of the production plan in the system degradation. Therefore, in each sub-period \( j \) of a period \( k \), the failure rate \( (t) \) is adopted according to the production rate \( \sum_{i=1}^{n} \).

Thus, the expression of the failure rate depending on time and production rate can be written as follows:

\[ \phi(t) \frac{\lambda}{\mu} \]

After simplification, (12) becomes:

**Lemma 2:**
failures can be expressed as follows:

Replacing

and defined by:

equation:

the average number of failures during $T$ is expressed by the equation:

$$
\phi_s = \int f^T \phi_s dr
$$

(13)

The difference with this study relies on the use of operating conditions that vary over time.

We know that different periods of maintenance are equal and defined by:

$$
\phi^T = (\phi^T ! \phi^T) ! !
$$

(14)

However, the average number of failures is defined by:

$$
\phi^T (\phi^T ! \phi^T) ! !
$$

(15)

Therefore, the average number of failures during the interval $[(\phi^T ! \phi^T) ! !] ! !]$ can be expressed as follows:

$$
\phi^T (\phi^T ! \phi^T) ! !
$$

(16)

Replacing $\phi^T (\phi^T ! \phi^T) ! !$ by its expression described in Lemma 2, in (16), the expression of average number of failures can be expressed as follows:

C. The expression of average number of failures:

Generally, in the case of maintenance with minimal repairs, the average number of failures is expressed for a defined duration and under operating conditions that are assumed to be constant over time. Under these assumptions, the average number of failures during $T$ is expressed by the equation:

$$
\phi^T = \int f^T \phi_s dr
$$

We recall that the objective of this section is to determine the optimal period of preventive maintenance actions that minimizes the total cost of maintenance. Therefore, we have to find the two variables $(! ! ! ! !)!!$ that minimize the expected total cost of maintenance described as:

$$
\phi^T (\phi^T ! \phi^T) ! !
$$

(17)

V. NUMERICAL EXAMPLE

A simple example of a hypothetical company that produces three different kinds of goods is considered now. Such products are strongly influenced by the fluctuation of demands, and the inventory level is not perfectly known due to several factors such. The manager desires to obtain a plan that minimizes the total cost of production and storage then the optimal period of preventive maintenance that minimizes the total cost of maintenance. The main firm’s data are listed in Table 1. The planning horizon is two years with monthly discrete time. The length of periods $t$ is equal to 3 months. The length of sub-periods $t^*$ is equal to 1 month. The standard deviation of demand of product $i$, is the same for all periods, $\sigma_i = \sigma_i (d_i)$. The customer service level is considered the same for the three products, $\gamma_i$, is equal to 0.80. The objective is to analyze the behavior of the process when the goal fixed by the manager is to satisfy demand at least 80%.

• The data relating to production:
To obtain the demands described in table 2, we have based on the follows standard deviation of demands: $\sigma(\lambda_i)$ ! $\sigma(\mu_i)$ ! $\sigma(\eta_i)$ ! $\sigma(\psi_i)$ ! $\sigma(\phi_i)$ ! $\sigma(\omega_i)$!

<table>
<thead>
<tr>
<th>Trimester1</th>
<th>Trimester1</th>
<th>Trimester1</th>
<th>Trimester1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product1</td>
<td>15</td>
<td>16</td>
<td>15</td>
</tr>
<tr>
<td>Product2</td>
<td>25</td>
<td>26</td>
<td>24</td>
</tr>
<tr>
<td>Product3</td>
<td>20</td>
<td>19</td>
<td>21</td>
</tr>
</tbody>
</table>

**TABLE 1.** FIRM’S DATA

<table>
<thead>
<tr>
<th>Data</th>
<th>Product1</th>
<th>Product2</th>
<th>Product3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production costs (Cp(i))</td>
<td>1.5</td>
<td>2.4</td>
<td>1.2</td>
</tr>
<tr>
<td>Inventory costs (Cs(i))</td>
<td>0.3</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>Initial Inventory (S(i,0))</td>
<td>3</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>The setup costs (St(i))</td>
<td>9</td>
<td>3.5</td>
<td>6.5</td>
</tr>
<tr>
<td>Customer’s satisfaction</td>
<td>92%</td>
<td>87%</td>
<td>90%</td>
</tr>
<tr>
<td>The maximum production rates (U_max)</td>
<td>20</td>
<td>30</td>
<td>25</td>
</tr>
</tbody>
</table>

Based on the analytical model described in subsection (3.B) and the data above, we obtain the following optimal production plan:

- The data relating to system reliability:

We use the optimal production plan obtained in table 2 in the expected total cost of maintenance $\psi(T)$ formulated in subsection (4.C). Concerning numerical example, the corrective and preventive maintenance costs are respectively $M_c=250\; mu$ and $M_p=150\; mu$. The failure rate of machine has a degradation law characterized by a Weibull distribution. The Weibull shape and scale parameters are respectively $\beta=2$ and $\lambda=5\; months$. However, the function of the nominal failure rate is expressed by:

$$\lambda(t) = \frac{1}{\beta} \left( \frac{t}{\lambda} \right)^{\beta-1} \exp \left( -\left( \frac{t}{\lambda} \right)^\beta \right)$$

- The obtained maintenance strategy:

Figure 2 shows the curve of the total cost of maintenance $\psi(T)$ according to number the intervention periods $(T)$. We conclude that the optimal period of intervention that minimizes the total cost of during the finite horizon $T=4\; months$.

**VI. CONCLUSION**

The key purpose of this study was to show the effect of the production rate variation on the optimal maintenance strategy. A stochastic production planning and maintenance scheduling problem was investigated under the assumption of a single machine producing several products. The machine is subjected to randomly failure. A minimal repair is practiced at every failure. In order to reduce the failure frequency, preventive maintenance actions are scheduling to the manufacturing system.

Firstly, given a satisfaction customers level and a randomly demands, we have formulated and solved a stochastic production problem in order to obtain an optimal production plan. Secondly, using the obtained production plan in the maintenance problem formulation, we established an economical scheduling in which we take into account the influence of the production rate on the system degradation.

**REFERENCES**


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